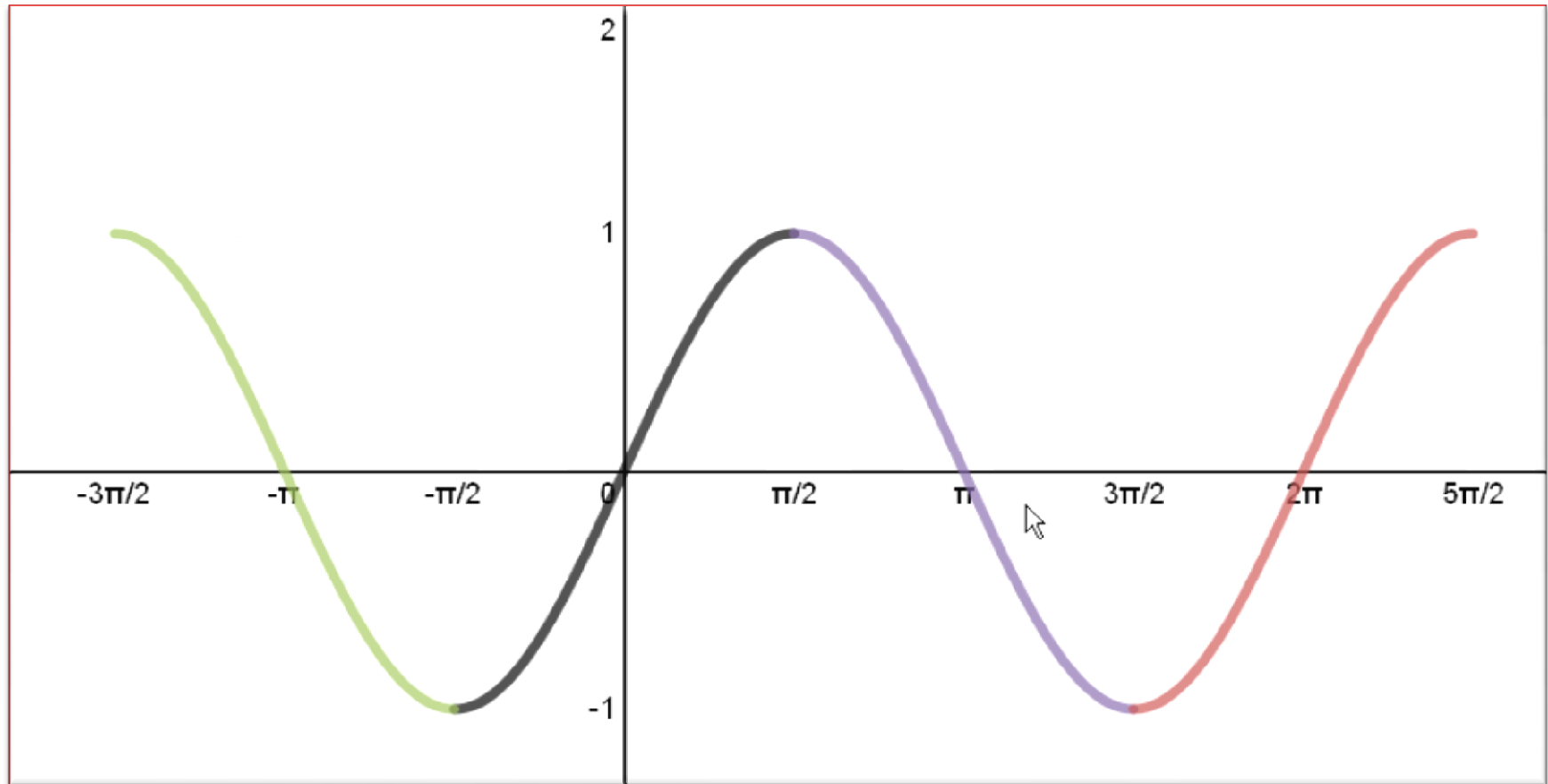


Inverse Trigonometric Functions

Principal Value Branch

$$f(x) = \sin(x)$$



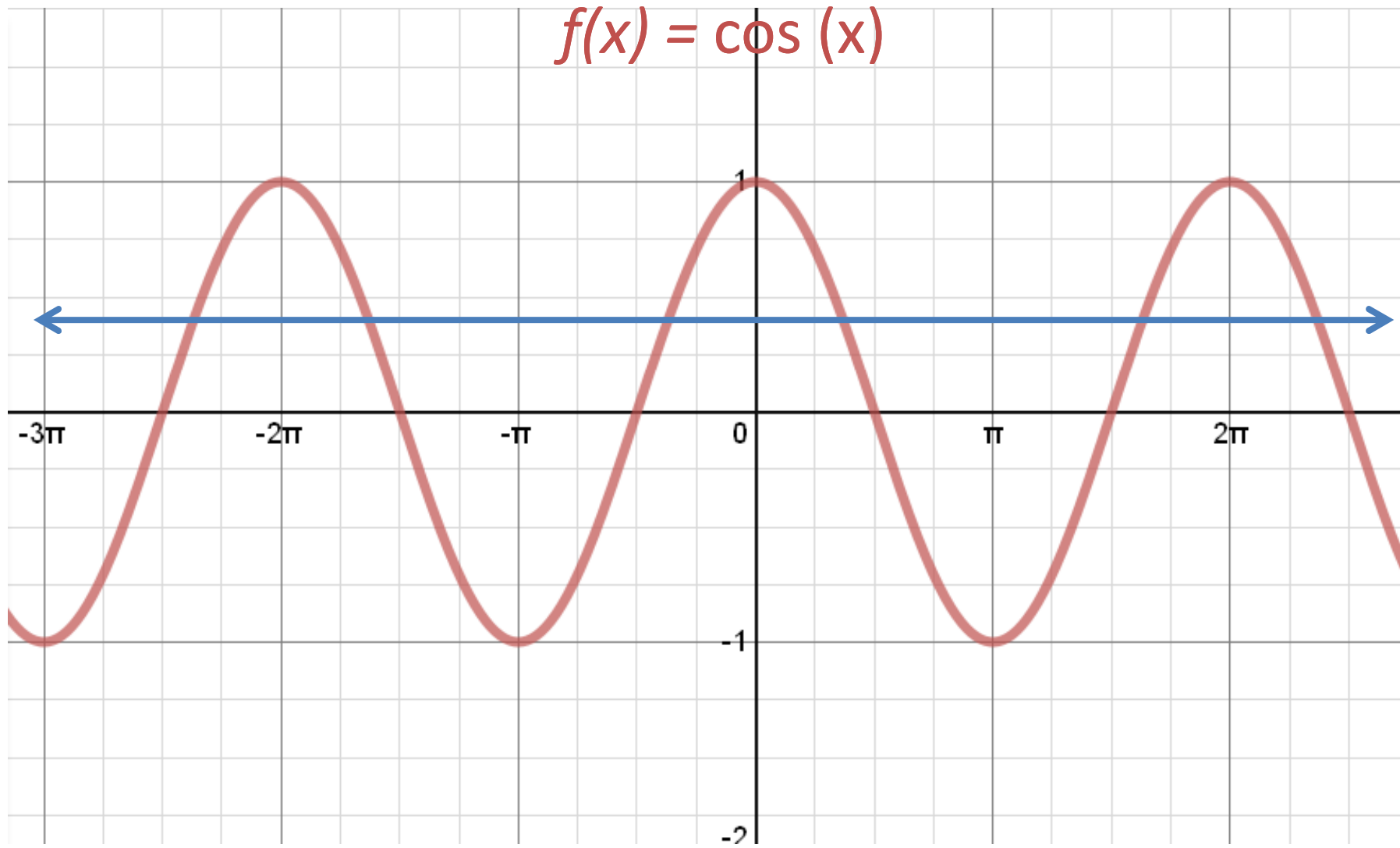
We define,

$$\sin^{-1} : [-1, 1] \rightarrow \left[-\frac{\pi}{2}, \frac{\pi}{2} \right]$$

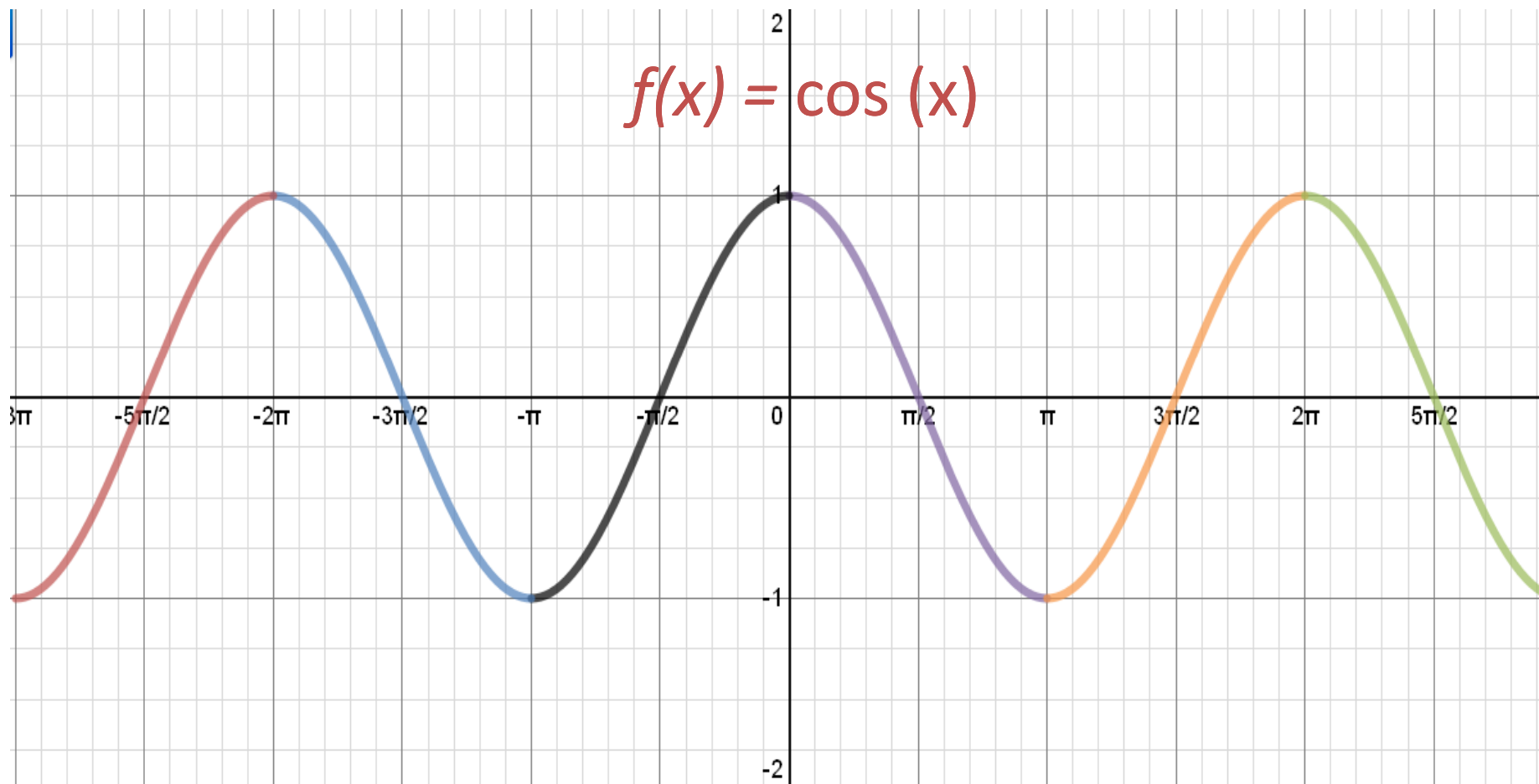
$$\sin^{-1}(x) = y \text{ if } x = \sin y$$

This branch is called the principal value branch.

$$f(x) = \cos(x)$$



$$f(x) = \cos(x)$$



$$\text{cosine} : [0, \pi] \rightarrow [-1, 1]$$

$$\text{cosine} : [\pi, 2\pi] \rightarrow [-1, 1]$$

$$\text{cosine} : [-\pi, 0] \rightarrow [-1, 1]$$

is one - one and onto

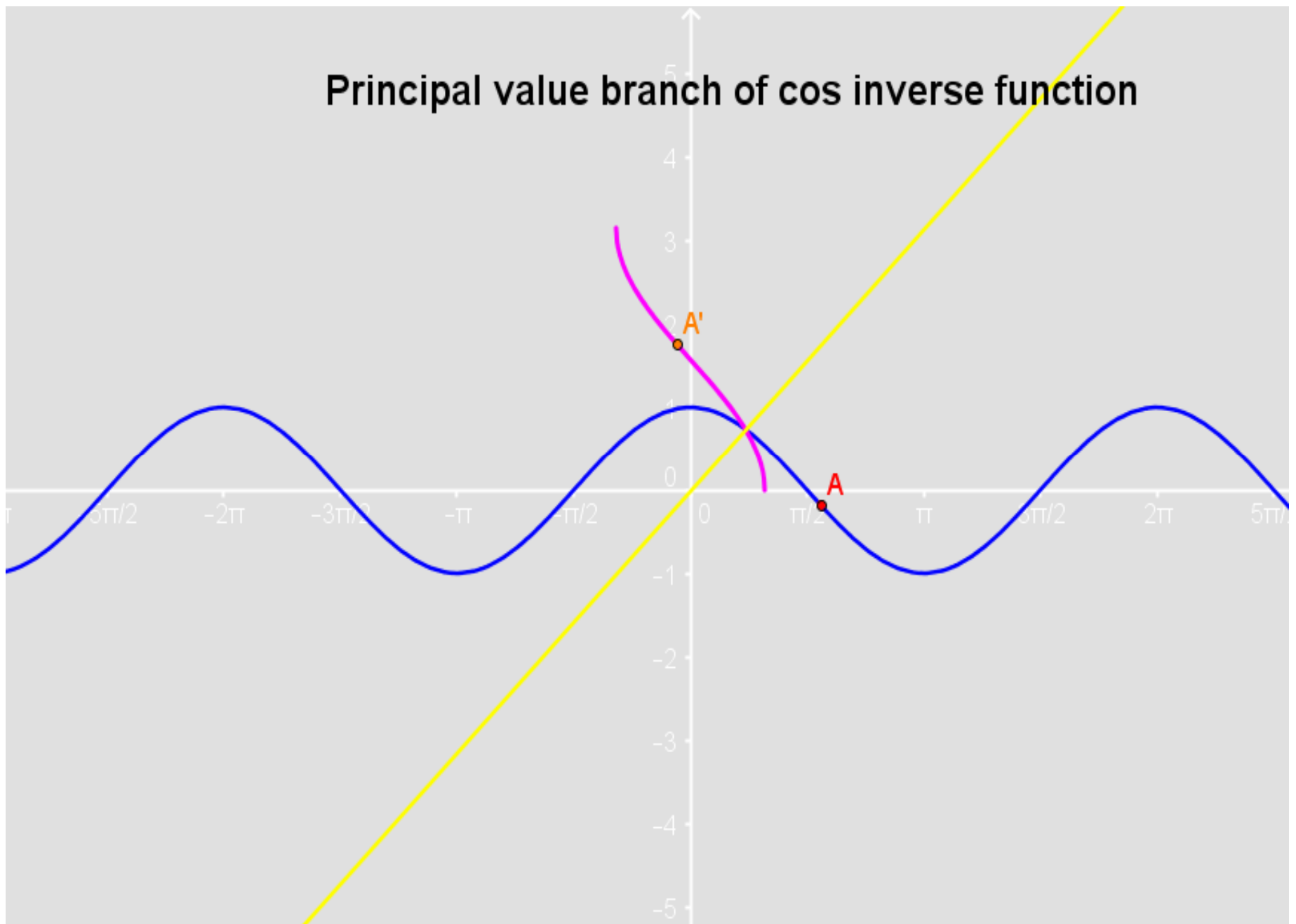
We define,

$$\cos^{-1} : [-1, 1] \rightarrow [0, \pi]$$

$$\cos^{-1}(x) = y \text{ if } x = \cos y$$

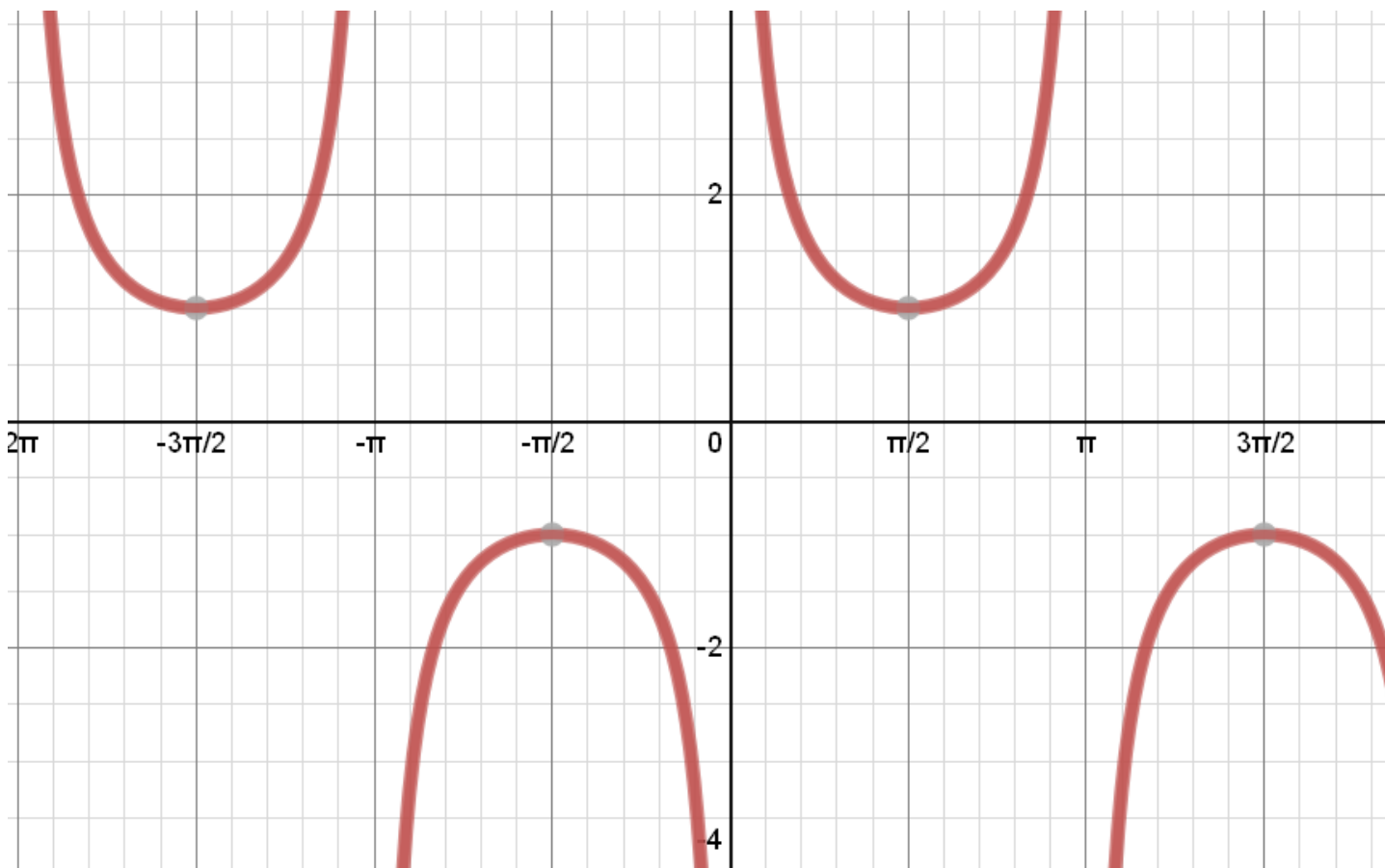
This branch is called the principal value branch.

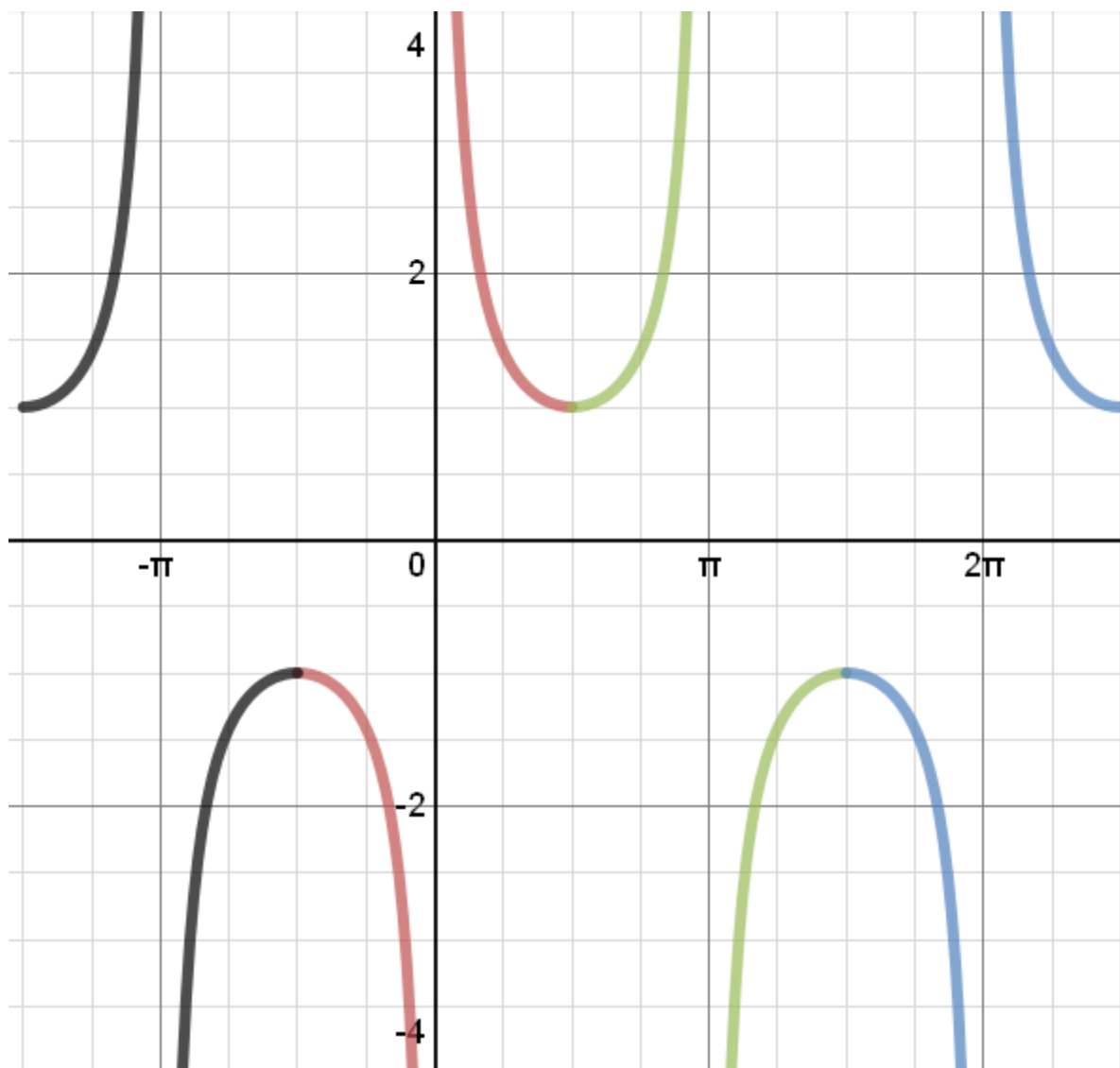
Principal value branch of cos inverse function



Domain of cosec function = $\{x : x \in R, x \neq n\pi, n \in Z\}$

Range of cosec function = $\{y : y \in R, y \geq 1 \text{ or } y \leq -1\}$
 $= R - (-1, 1)$





$$\operatorname{cosec} : \left[\frac{-\pi}{2}, \frac{\pi}{2} \right] - \{0\} \rightarrow R - (-1, 1)$$

$$\operatorname{cosec} : \left[\frac{\pi}{2}, \frac{3\pi}{2} \right] - \{\pi\} \rightarrow R - (-1, 1)$$

$$\operatorname{cosec} : \left[\frac{-3\pi}{2}, \frac{-\pi}{2} \right] - \{-\pi\} \rightarrow R - (-1, 1)$$

is one - one and onto

We define,

$$\cos ec^{-1} : R - (-1,1) \rightarrow \left[-\frac{\pi}{2}, \frac{\pi}{2} \right] - \{0\}$$

$$\cos ec^{-1}(x) = y \text{ if } x = \cos ec y$$

This branch is called the principal value branch.

Principal value branches of inverse trigonometric functions

$$\sin^{-1} : [-1, 1] \rightarrow \left[-\frac{\pi}{2}, \frac{\pi}{2} \right]$$

$$\cos^{-1} : [-1, 1] \rightarrow [0, \pi]$$

$$\operatorname{cosec}^{-1} : R - (-1, 1) \rightarrow \left[-\frac{\pi}{2}, \frac{\pi}{2} \right] - \{0\}$$

$$\sec^{-1} : R - (-1, 1) \rightarrow [0, \pi] - \left\{ \frac{\pi}{2} \right\}$$

$$\tan^{-1} : R \rightarrow \left(-\frac{\pi}{2}, \frac{\pi}{2} \right)$$

$$\cot^{-1} : R \rightarrow (0, \pi)$$

Q : Find the value of : $\sec^{-1}(-\sqrt{2}) + \cot^{-1}(\sqrt{3}) - \tan^{-1}(-1)$

Solution :

$$\text{Let } y = \sec^{-1}(-\sqrt{2})$$

$$\Rightarrow \sec y = -\sqrt{2}$$

$$\text{Principal value of } y \in [0, \pi] - \left\{ \frac{\pi}{2} \right\}$$

$$\therefore y = \frac{3\pi}{4}$$

$$\text{i.e. } \sec^{-1}(-\sqrt{2}) = \frac{3\pi}{4}$$

Q : Find the value of : $\sec^{-1}(-\sqrt{2}) + \cot^{-1}(\sqrt{3}) - \tan^{-1}(-1)$

Solution :

$$\sec^{-1}(-\sqrt{2}) = \frac{3\pi}{4}$$

$$\cot^{-1}(\sqrt{3}) = \frac{\pi}{6}$$

$$\tan^{-1}(-1) = \frac{-\pi}{4}$$

$$\therefore \sec^{-1}(-\sqrt{2}) + \cot^{-1}(\sqrt{3}) - \tan^{-1}(-1) = \frac{7\pi}{6}$$